*Rivista Italiana di Economia Demografia e Statistica* *Volume LXVI n. 2 Aprile-Giugno 2012*

# THE EVOLUTION OF OPTIMIZATION MODELS SUPPORTING MOBILITY POLICIES AND TERRITORY DEVELOPMENT

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## 1 Introduction

In this paper four major milestones in the development of optimization models are identified in the perspective of policy decision making for mobility and land development. These milestones definition is arbitrary and is motivated only with the purpose of helping the reader in following the contribution of mathematical programming techniques into the decision-making tools that have been, and still are, practically adopted in strategic and planning territory management. The milestones defined will be referred as: Linear Programming, Integer and Mixed Integer Programming, Integration of Optimization Models in Real-Time ICT Architectures, Optimization and Business Analytics.

We introduce the reader into the evolution of optimization models so that the contribution of the new outcomes in relation to applications in transportation and mobility appears more evident.

The paper is organized as follows. In section 2 the main characteristics of Linear Programming, which are at the base of any optimization model are recalled and the evolution seen from the application point of view is sketched. Section 3 underlines the difference between linear programming and integer programming and some example of integer optimization problems on graphs are given. In Section 4 the role of optimization model with respect to Information and Communication Technology development for transportation is briefly described. Finally, in Section 5 is presented an example of application of optimization for mobile services for tourism in metropolitan areas.

The reader should relay on the references for further investigations.

## 2 Linear Programming and the Management of Continuous Resources

Linear programming allows to model an optimization problems by means of continuous variables that may represent, in general, a continuous divisible resource as, for instance, the quantity of flow in an arc of a network or the portion of land that should be cultivated with certain grain. In general, in Linear Programming, we want to find an assignment of the variables that maximize (or minimize) a given linear function representing, for instance, profits (or costs). Such assignments must obey to a number of constraints. Clearly, the flow in an arc cannot exceed the capacity of the arc or the sum of assigned portions of lands cannot be grater than the area of the region under study.

See the model below reported for illustrative purposes, where the decision variable $x_i$ represents the quantity of a (continuously divisible) kind of good *i* to be produced, and coefficients $a_{ij}$ the quantity of raw material of kind *j* required for one unit of good *i*, and $b_j$ the total amount of raw material j available. The $c_i$ coefficients represent the profit that can be obtained by a unit of good *i*. Solving the model permits to find the best (more profitable) quantity of each good to be produced in order to maximize profit while respecting the constraints on the total availability of each resource ($b_j$).

*minimize*

$$z = c_1 x_1 + \ldots + c_n x_n = \sum_{j=1}^{n} c_j x_j. \tag{1}$$

*such that*

$$\begin{array}{llll} a_{11}x_1 + & \ldots & +a_{1n}x_n & \leq b_1 \\ a_{21}x_1 + & \ldots & +a_{2n}x_n & \leq b_2 \\ \vdots & \ldots & \vdots & \vdots \\ a_{m1}x_1 + & \ldots & +a_{mn}x_n & \leq b_m. \end{array} \tag{2}$$

$$x_i \geq 0, \quad i = 1, \ldots, n.$$

As can be noticed, both objective and constraints are linear functions. Linear programming is a very important class of problems, both for modelling capabilities and for the algorithmic implications. Linear programming has so many applications that we have to refer to the books in the references. The basic modelling result can be seen as in this way: any practical interesting configuration of the decision variables (corresponding to a specific real scenario) is one of the vertices of a convex polyhedron in $R^n$ and the number of these vertices (although extremely) large is finite (see Figure 1). Assume the resource to be assigned is water for the agriculture in different fields in a given area. The optimal water allocation on the different fields (i.e. maximizing a give linear economic or function,) is one of the vertices of the polyhedron. This remains true for any objective function also if obtained as a linear combination of different criteria (i.e. any compromise between profits and environmental impact).

**Figure 1** – *An illustrative representation of the solution space of an LP program.*

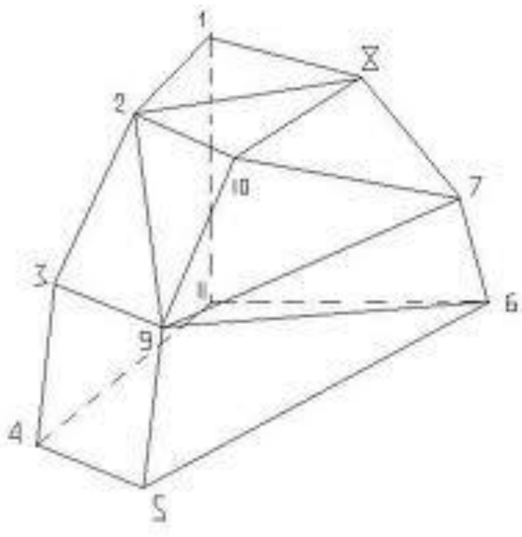
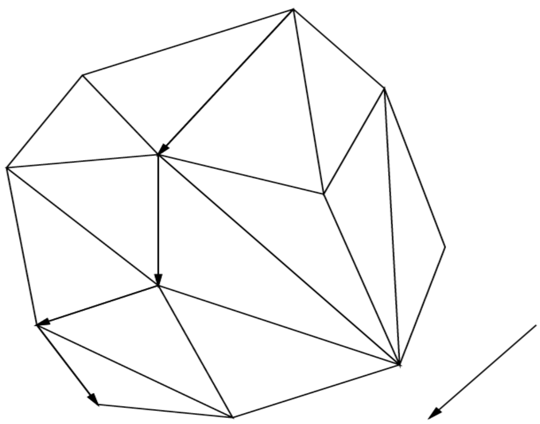



**Figure 2** – *An illustrative representation the steps of the simplex algorithm toward the optimal solution.*



From an algorithmic point-of-view, having to search only for vertices has a great implication. The simplex, that moves from one vertex to an adjacent and better one, was proposed in the forties (soon after the war, and was motivated by military applications, Dantzig 1947) and, although it has performed very well in practice, is known to run in exponential time in the worst-case. The first polynomial-time algorithm, the ellipsoid algorithm, was only discovered at the end of the seventies (Kachian 1979). Karmarkar's algorithm in the mid-eighties lead to very active research in the area of interior-point methods for linear programming. We recall some of the numerous variations of interior-point methods in class (Dikin 1967), (Frisch 1977).

A first application of optimization algorithm used in practice in Italy in the cast iron production in the sixties reduced by 50% the cost. The application of the simplex method was done using an electronic desk calculator to support each step of the algorithm by two people in time measured in weeks.

Currently, the improvements, both on algorithms and in computer computational power allows to solve problems with 1 million variables in tenths of seconds.

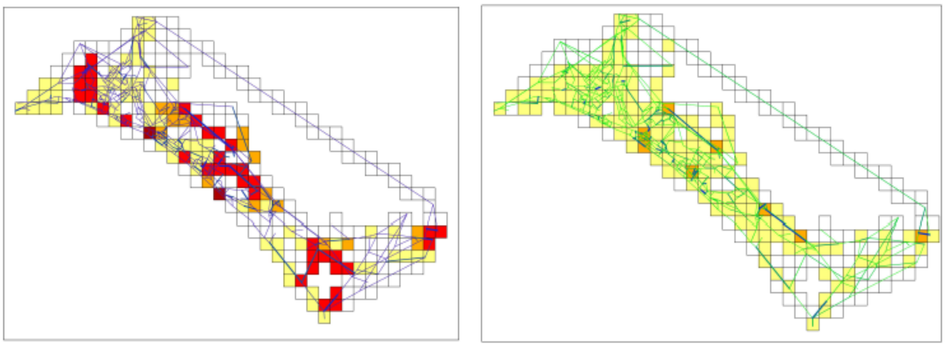
For a more recent application of linear programming in the are of land planning we mention the paper of Alampi Sottini et al. 2008, where an LP program is designed and solve to simulate both a more profitable and sustainable use of land in the area of Mugello (Tuscany, Italy) taking into account several constraints like the maximum of agriculture surface available, the surface and aliments for animals, the maximum number of family manpower, the agricultural and zootechnical productions. Solution results visualized by means of a Geographical Information System permit to the decision-maker to analyse very complex scenarios.

A further example is in equitable flow assignment on networks (see Dell'Olmo 2011). In this case, we have to assign flows on arcs of the network so to satisfying the demand of goods transportation on different origin-destination points with the double objective of minimizing both transportation cost and the impact of traffic in different areas.

The example of the application of a Linear Programming model to flow assignment for the network of Salerno is given in the next Figure. The color of an area represents the traffic volume in that area (red = high, yellow = low). As can be seen, in the right picture the optimized flow assignment has the same cost, but with a more equitable impact on the areas than the case of the picture to the left.

Linear programming capabilities are now available in commercial software tools to anyone interested in practical use of this kind of optimization models, only in specific cases research activity is devoted in improving solution algorithms.

**Figure 3** – *A grid of uniform cells overlapped to the underlying road transportation network of the city of Salerno, Italy.*



## 3 Integer Programming, Logistics and Freight Distribution

In the early seventies the classes P and NP (Garey and Jhonson 1979) were defined, it was observed that a large class of optimization problems were equally difficult to solve and for no one of them polynomial time algorithm (like for the case of linear programming) exists. Routing and scheduling problems, which are the basic ingredients of distribution logistic management, belong (unfortunately) to this class. For these problems, decision variables must assume integers values and, in a wide range of cases, variable values are only 0 or 1. In many routing problems, for instance, a value of 1 of a variable means the corresponding arc is part of the solution, 0 means the opposite. As in practice one wants to know if an arc is in his paths or not, fractional values of decision variables have very little meaning,

The classical problem of this kind is the well known Travelling Salesman Problem (TSP). The TSP is so defined: Given a complete undirected graph *G=(V, E)* that has nonnegative integer cost *c(u, v)* associated with each edge *(u, v)* in *E*, the problem is to find an hamiltonian cycle (tour that visit each node only once) of *G* with minimum cost.

Here, the basic difficulty, for which efficient algorithms like those for linear programming cannot be applied, is that the solution set is not represented by the vertices of a polyhedron, but by integer points in a multidimensional solution space.

No general efficient solution method exists at this moment for exact solutions of this type of problems and a great effort has been done in designing specific solution procedure for problem classes.

**Figure 4** – *Example of TSP solutions of value 62 and 48 respectively.*

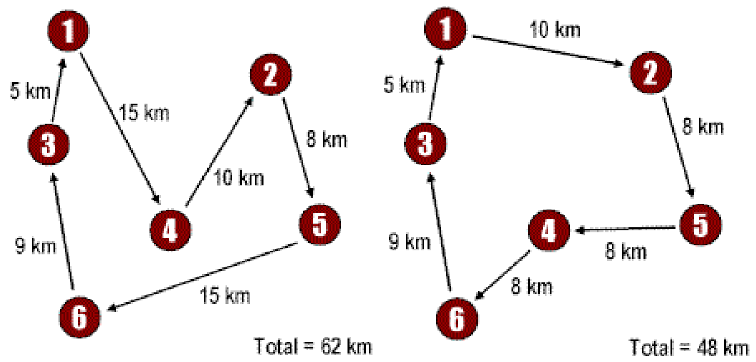
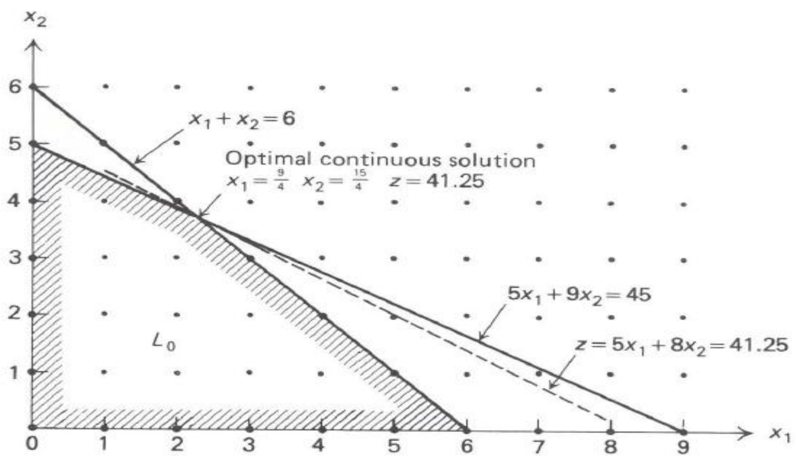


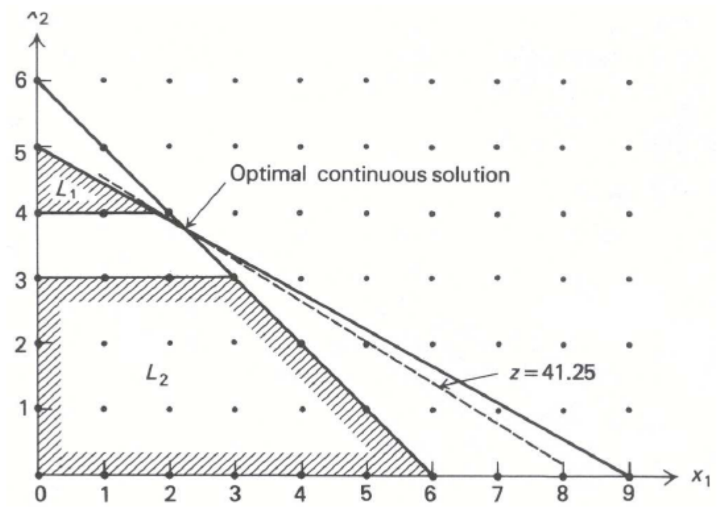

**Fig. 5** – *The solution space of an integer programming is represented by the dots contained in the polyhedron.*




The main contribution of optimization methods from the late seventies up to now is to having provided a number of different solution approaches to solve computationally difficult problems of this kind.

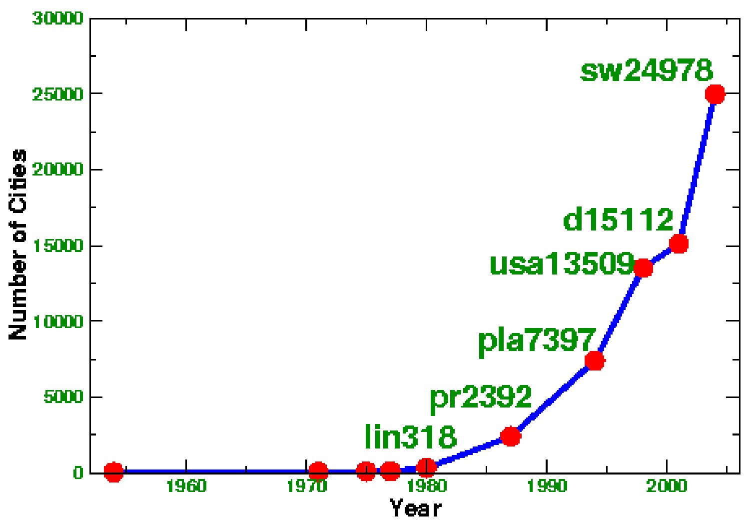
Algorithms like Dynamic Programming, Branch and Bound, Branch and Cut (see references book integer programming) have been proposed to find exact solutions to computationally intractable problems (see picture below that illustrates the way branch and bound partitions the solution space).

**Figure 6** – *The partition of the solution space operated by branch and bound algorithm.*




To give an idea of progresses in algorithm design in the last decades see the following picture where the size (number of cities) of the TSP instances solved is reported on the *y* axis over the years.

**Figure 7** – *The size of TSP instances solved over the years.*



Beyond the performances in test problem, it has to be noticed that the same techniques are implemented in other routing models (for instance Multi-Depot Vehicle Routing Models, Toth 2002) that are fundamental tools for a competitive distribution logistic system.

Currently, a significant effort of the research activity in this area is devoted in finding better solution methods (i.e. capable of solving real life large instances) and algorithms for specific problems that take into account a number of operational constraints (like time windows, priorities and the like). See Golden 2008 for recent advances in this area.

## 4 Real-Time Optimization and Services on Transportation Networks

In the last decade, transportation networks have been equipped with a number of different devices (traffic sensors, cameras, on board GPS, etc.) which changed significantly the way optimization models are used in traffic control. Especially in metropolitan areas, a complex communication network gathers data from different sources into a central control system where several decisions can be taken regarding traffic lights control, Variable Message Signs (VMS) implementation and further services for public and private transportation (see the Figure below).

**Figure 8** – *Different sources and devices to collect real time data.*

In this case, optimization models are components of larger software architectures and very often are fed with real time data with the purpose of having real-time decisions.

For example, when traffic congestion slow down traffic flow on a given road, routing algorithms must be able to providing alternative paths in tenth of seconds ensuring that the whole network reaches a new acceptable state of stochastic equilibrium.

A further example of application is when distribution services accept on line (real time) requests of new pick-up and delivery. In those cases, the fleet paths must be dynamically adapted to incoming demand and on board equipment is necessary to advisor the driver for the change. For this class of models execution time must be kept as small as possible and a number of additional constraints must be taken into account also additional requirements like inventory (see Favaretto et al. 1998) for an example.

## 5 Mobile Services for Tourism

Following what has been described in the previous section, we present here an application of optimization models to support the mobility of a specific class of users, that is tourist in large metropolitan areas.
In this case, available time for the visit is always limited and one wants to visit as many interesting places as possible. Clearly there are several ways to approximate the utility function of a specific tourist (knowing his/her characteristics and preferences), and an example of objective function is given next:

$$\max(Z) = \sum_j S_j^k y_j \quad + \sum_i \sum_j S_{ij}^k y_{ij} \tag{3}$$

Here the variable $y_j$ is 1 if the place is visited and, similarly, variable $y_{ij}$ is 1 if the arc connecting place *i* to place *j* is chosen, and 0 otherwise. The coefficient $S^k{}_j$ and $S^k{}_{ij}$ can be seen as fuzzy estimates of the marginal value for the tourist *k* in visiting location *i* or travelling in the road connecting *i* to *j* respectively.

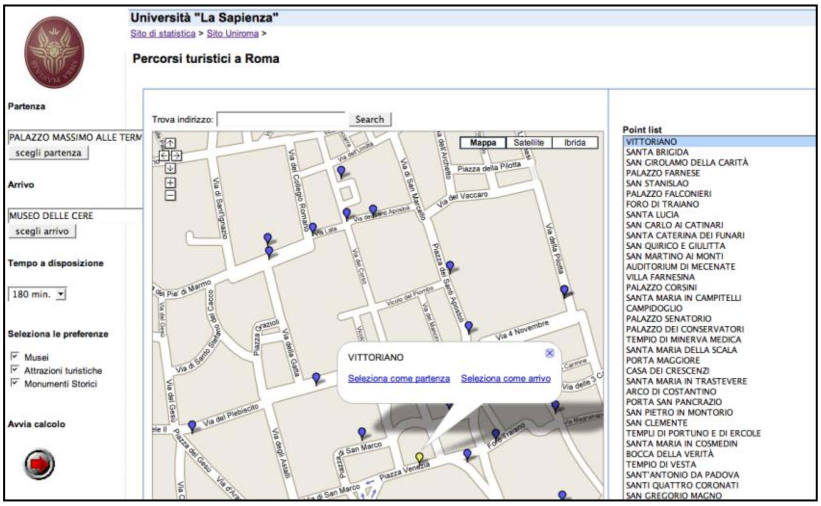
Time constraints can be represented as follows:

$$t_{max} \leq \sum_j t_j^k y_j \quad + \sum_i \sum_j t_{ij}^k y_{ij} \tag{4}$$

where $t^k{}_j$ is an estimation of the time required that tourist *k* requires to visit location *j* and $t^k{}_{ij}$ ; $t_{max}$ is the total time available.

Further constraints are necessary to complete the model. As for the previous section, the optimization model is only a part of a service system that could be presented as a web application (see the Figure below).

**Figure 9** – *A web based application where a tourist define start and end locations of his/her visit.*




The way the service works could be sketched as follows. To the user it is presented a geographic representation of the area he/her wishes to visit (with interesting location highlighted. The user denotes:

- Main interests (museum, historical monuments, etc.)
- Time available for the trip
- Starting location
- Ending location

The system computes the walking path that maximizes the user score of the visit (that takes into account number and interest of the sites and other factors) within the available time.

Here the role of optimization models are very clear on large networks in cities that offer a variety of alternatives and some of them may be very interesting but not so well known like for the case of Rome (see the picture below with the Rome network).

**Figure 10** – *The graph of Rome with 1092 nodes, 4468 arcs and 226 places of interest.*

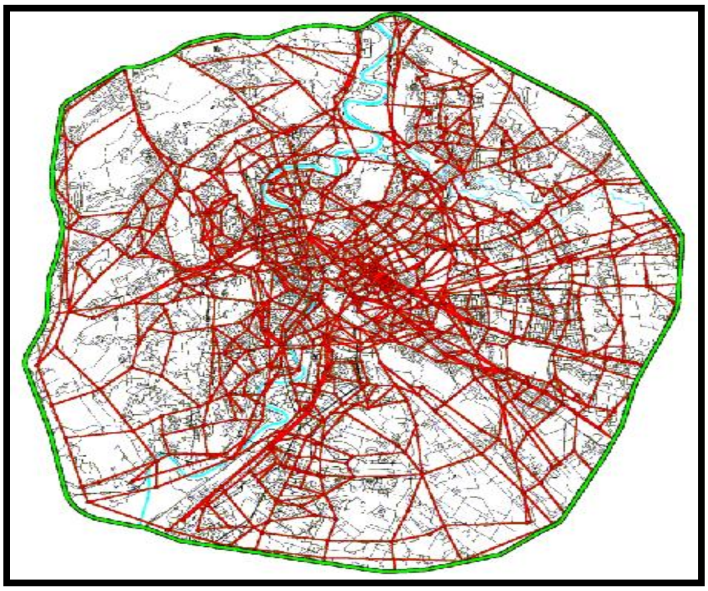
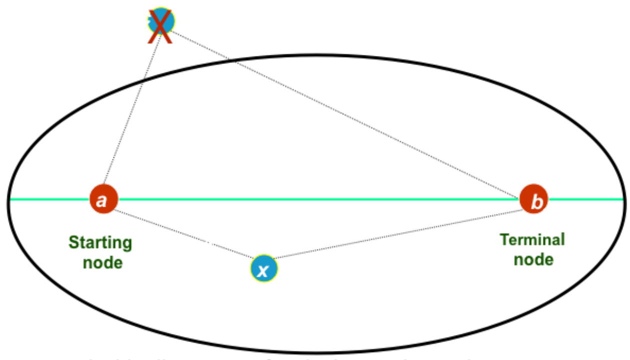


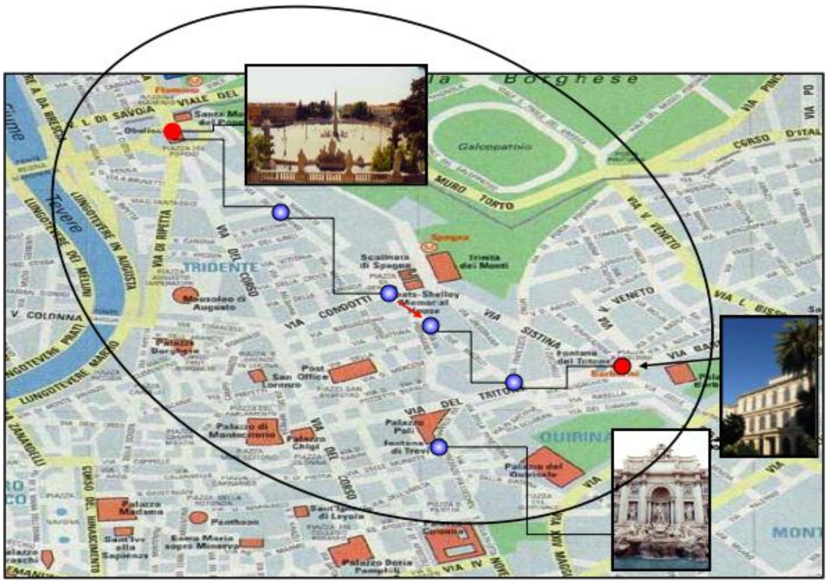
**Figure 11** – *The perimeter of places of interest so that sum of distances a to x and x to b is equal*




In order to limit the search to places that can be reached within the time limit, an ellipse is defined with first and second focus the starting and ending point of the visit.

The ellipse radius is computed calculating the maximum distance that can be travel by walking in the maximum available time.

**Figure 12** – *The graph of Rome with 1092 nodes, 4468 arcs and 226 places of interest.*




Thus, the main idea is that any of the places that is located within the ellipse could be added to the start-ending point of the visit, so we can select the one with maximum interest. From the computational point of view, however, the complete algorithm is not so straightforward are requires several steps to reach the optimal solution.

In each step a new location is inserted and a new ellipse is designed using the new locations as focus. The iterations continue finding a new ellipse and a new location to be added until no more places can be inserted to the path because of the time limit constraint (see the next picture).

Although important, the optimization algorithm to furnish a practical service to an end-user (tourist) requires a quite complex architecture. In the next figure we present the basic components of a possible architecture.

It should be clear that a lot of data from different sources should be integrated for the practical realization of this kind of service. For instance, time to visit locations as museums or churches has to be known in advance and coordinated with public services transportation availability and estimated travel times. Similarly, with other services like restaurants, cafés, and shops open hours has to be acquired.

**Figure 13** – *An illustrative example of the application to maximize tourist utility function.*

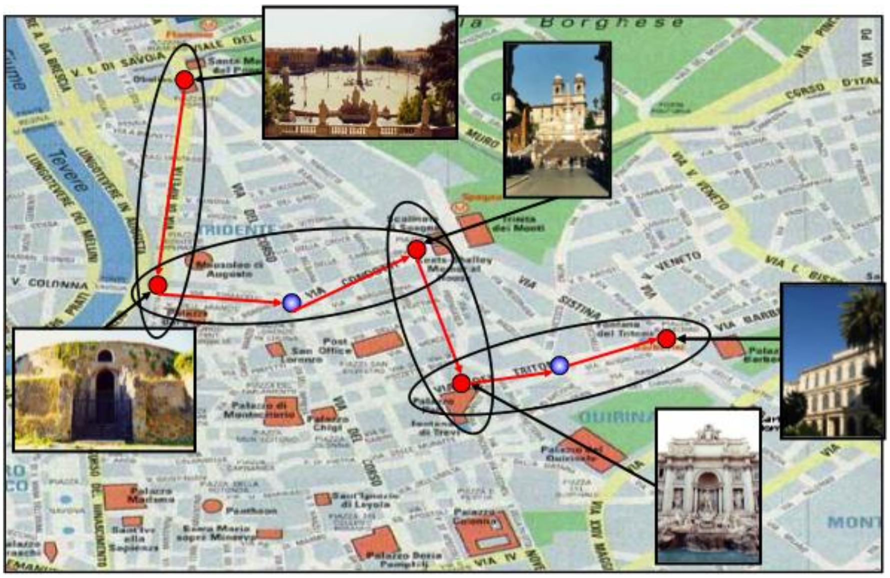
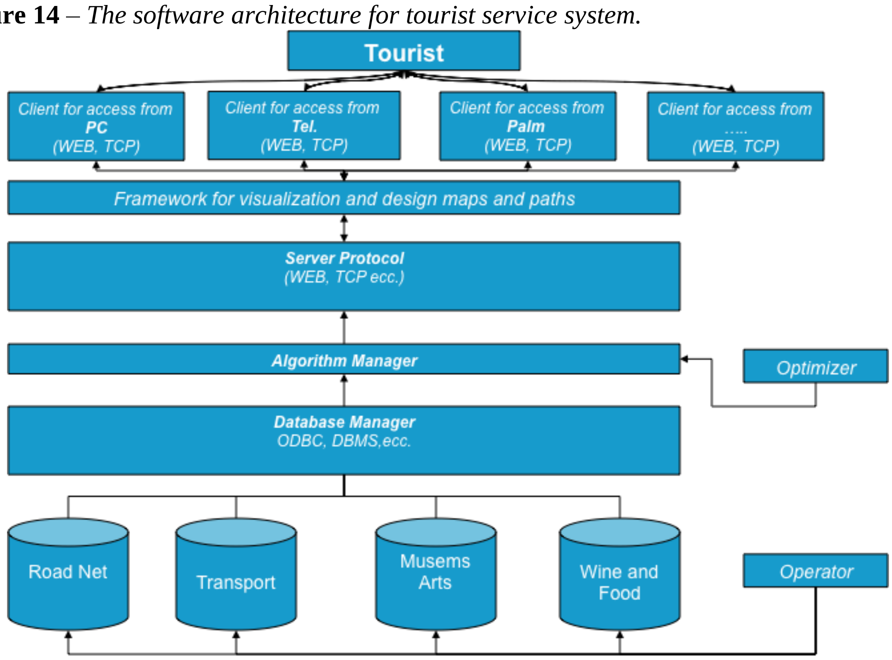


**Figure 14** – *The software architecture for tourist service system.*



## Acknowledgements

The author wish to thank Umberto Ferraro Petrillo, Nicoletta Ricciardi and Gianni Storchi for their contribution in the tourist service system.

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## SUMMARY

# The evolution of optimization models supporting mobility policies and territory development

The evolution of optimization techniques if often hidden in currently used decision making systems and advisory tools that support decision making at policy level. No matter how much system integration and availability of data pushed the innovation, in many cases, like distribution logistic, no practical results could be achieved without the improvement in algorithms design we have seen in the last decades. Such algorithms, integrated in a Geographical Information System to support land development plans, or in a smartphone to give personalized mobile services to tourists, make it possible to obtain answers to complex decision problems in reasonable time, not because of computational power of the processors, but for the efficiency of the solution methods. In the paper, we attempt to follow the main steps of optimization algorithms giving some examples of their impact in managing problems related to mobility and territory.

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