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DECISION MAKING AND SAINT PETERSBURG PARADOX: FOCUSING ON HEURISTIC PARAMETERS, CONSIDERING THE NON-ERGODIC CONTEXT AND THE GAMBLING RISKS

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1. Introduction

In 1713 Nicolaus I Bernoulli, in a correspondence with Pierre Rémond de Montmort, first identified a particular lottery game producing infinite expected gain[1]. However, the appellation of this mechanism as *Saint Petersburg Paradox* is coming from Daniel Bernoulli's contribution, entitled *Specimen theoriae novae de mensura sortis* (1737) in the *Commentaries of the Imperial Academy of Science of Saint Petersburg*. In this publication, Daniel Bernoulli clearly shows the intent of measuring the risk (*mensura sortis*) starting from some case studies and proposes a solution of the paradox stated by Nicolaus. Actually, the scope was to challenge the theoretical predominant paradigm of the expected value used for the risk assessment. The risk assessment analysis connected to the paradox is often forgotten because the attention is mainly focused on finding some convergence of the expected value.

2. Scope of the formalisation and decision making

Since theories are built to help us with the practical decisions, most readers may be not necessarily interested in pure mathematical demonstrations dealing with the problem of infinity which, furthermore, would have no impact on the limited human life cycle. In fact, it would be worthwhile to focus on some basic parameters in order to understand the risks of decision biases coming from some theoretical results. Moreover, the "tendency to infinity" of the expected value can be "balanced" with a counter-formal heuristic approach, which can underline the illusory possibility of the infinite gain emerging of the paradox. The following

[1] "*Peter tosses a coin and continues to do so until it should land 'heads' when it comes to the ground. He agrees to give Paul one ducat if he gets 'heads' on the very first throw, two ducats if he gets it on the second, four if on the third, eight if on the fourth, and so on, so that with each additional throw the number of ducats he must pay is doubled. Suppose we seek to determine the value of Paul's expectation*". Letter dated 9 September 1713 to P. de Montmort (correspondence of N. Bernoulli on St. Petersburg Game - Translated by R. J. Pulskamp, Xavier Univ, Cincinnati, OH. Jan 1, 2013).

quote by A. E. Newton could be useful to make some reflections on the infinity and the semantic of it:

"*Who was it who said, "I hold the buying of more books than one can peradventure read, as nothing less than the soul's reaching towards infinity; which is the only thing that raises us above the beasts that perish?" Whoever it was, I agree with him".* (A. E. Newton, 1921. A magnificent farce and other diversions of a book collector)

When we talk about infinity, sometimes we simply intend a huge number of something (*more books that anyone can read;* because only in this way we could let our *soul reach the infinity*). This may imply many reflections, for instance: provided that we have a *soul* (and this would already be in fact our intrinsic infinite quality), why we should employ large amount of finite things in order to reach the infinity that we already possess? Why possess for a personal scope something that you could never read? Anyway, how do we use these books? Do we simply look and admire the covers? Do we leave them to future generations? And, in the end, if we just perish like the *beasts* he cited, what sense could all this have? Therefore, this quote also contains many paradoxes and, I would say, as much confusion as the St. Petersburg paradox does. In the case of A. E. Newton's quote, a rational observer would say that the most common reason for being a collector is a compulsive self-satisfying behaviour driven by the emotions connected with the ever new objects coming into his possessions. Now, we should try to detach a little bit from the pure theory and ask ourselves some core questions: why do we use formalisation? Is it always useful to reason with infinite perspective? What is the context of the study? Does it help us to make useful choices? And, finally, are mathematical axioms a dogma?

3. Saint Petersburg paradox

What is the paradox about? The game consists in tossing a coin. The consecutive occurrences of tail events will produce a gain which value will exponentially increase as long as tails continue to be consecutively generated by each toss [2]. The game ends when head occurs. If we formalise these procedures and make some calculations, we realize that the game generates an infinite expected gain [3]. According to the original formulation of the paradox (note 1) the EV would be:

$$E(X) = \sum_{k=1}^{n} 2^{k-1} \cdot 2^{-k} \qquad (1)$$

However, if we suppose that the gain after the first tail occurrence is *2* Euro (instead of 1) and that the gain after *n* tails in a row will be 2^n Euro (instead of 2^{n-1}), the substance of the paradox would not change much and we would follow and execute more smoothly some calculations. The expected value of the game will therefore be:

$$[2]\ E(X) = \sum_{k=1}^{n} 2^k \cdot 2^{-k} \quad \text{for } k \to \infty \text{ it will produce an infinite expected gain} \tag{2}$$

$$[3]\ E(X) = \sum_{k=1}^{\infty} 2^k \cdot 2^{-k} = \infty \tag{3}$$

4. A very practical answer to Saint Petersburg paradox

If we want to "monetarise" at a certain point, we should interrupt the game. Therefore, the number of tosses should became finite and this would already undermine the expected infinite value which, as empirically observed, is logarithmically diverging for a large number of repeated games (see a simulation in fig. 3). Moreover, the expected value is also a questionable parameter for forecasting the most probable outcome, especially when the most remunerative events are associated to lower and lower probabilities[2]. Moreover, even if the number of tries becomes finite, the expected value - as we will see hereafter - could represent a suboptimal reference for the decision making process. Actually, there is not a real paradox but only a fallacy in the choice of the model to describe the empirical case. In fact, the biases generating the paradox seem connected on how we calculate the expected value (EV). A practical example of how a decision can be biased if it is taken only on information coming from EV is the game show *Deal or No Deal*. At a certain point of the game the player receives a money offer (usually inferior to the EV calculated on the remaining prizes) for ending the game and renounce gambling for further higher available prizes. Since the choice is only based on the EV, the player should always renounce because of the unfair proposal. However, it could be sometimes wise to accept the offer despite its being inferior to EV because of the high dispersion of the value of the prizes and the great incertitude connected with their probability to occur[3]. Actually, the expected value formula may be the key of the paradox because it fails to give fair practical information on how to assess a very uncertain stochastic context. In fact, the distribution of probabilities connected to the payoff in the St. Petersburg games is very asymmetric and therefore it could not be sufficiently described by the mean

[2] In other words, the EV is only a theoretical reference but may not always be a good parameter for making the best choice especially if the game is a stochastic variable with infinite possible results, it presents a high risk of low payoff and it is repeated only a few times.

[3] For example, if the remaining prizes are 1000, 1500, 2000, 2500, 3000, 99000, and therefore the EV=18166, it could be wise to accept an offer of 9000 because, in this game without repetition, it would be very risky to continue gambling to reach the highest available price (the probability of getting the best prize would only be 16,6 % while the chances of ending with less than the offered amount is 83,4).

(in this case the EV would be undefined) as in the normal distributions. If instead of the mean (EV) we consider the median value[4] (which could more fairly reduce the noise of highly skewed distributions), a fair price to enter the game should be of very few Euro.

No reasonable gain could be expected (at least in a finite number of games) considering the high magnitude of uncertainty about the occurrences of highest gains and their capacity to cover (in case of repeated games) all the previous loss in order to obtain a reasonable positive payoff.

Bernoulli himself proposed a solution for the paradox, but his attempt to resolve it with a utility function was not a "real solution" to the paradox in itself, even if it is a valuable effort to approach a theoretical concept in an empirical context. In fact, Bernoulli proposed a utility function[5] that considers the player's expected utility as a natural logarithmic function of the expected payoff. In other words, utility does not scale linearly with the payoff value but is logarithmically decreasing.

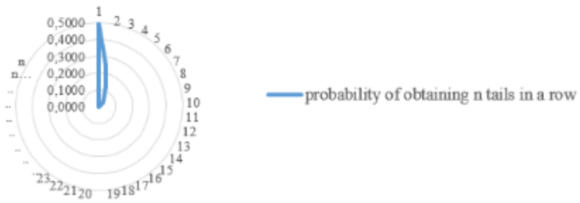
$$EU(X) = \sum_{k=1}^{\infty} \ln 2^k \cdot 2^{-k} \tag{4}$$

The problem with this function, beside the choice of its characteristics based on the psychological factors, is that we could always conceive another paradox that would not be explained by the ad hoc built function. If we suppose the payoff is e^{2^k}, the function is not resolving the paradox anymore[6].

$$EU(X) = \sum_{k=1}^{\infty} \ln e^{2^k} \cdot 2^{-k} = \infty \tag{5}$$

If we develop the matrix of probability connected with the increasing win associated with the consecutive tails occurrences, we would obtain a rapidly decreasing plot associated with increasing consecutive winning events (Fig.1).

Fig.1 - Probability connected to increasing win for consecutive tails occurrences



[4] In the case of the previous note example about the show *Deal or No deal*, the median (2250) could represent a better parameter for evaluating the convenience of the offer.

[5] V*alue must not be based on the price, but on the utility it yields. A gain of one thousand ducats is more significant to the pauper than to a rich man though both gain the same"* (D. Bernoulli 1788).

[6] Maybe, since the economic theory mainly aims at teaching us how to behave in uncertain situations in order to make the best possible choices, we could be more interested in a finite version of the game.

5. Some heuristic hints

If we consider the sum of the probabilities connected to the payoff emerging from all the possible series of tails in a row[7], we would have:

$$F(G) = \sum_{k=1}^{\infty} 2^{-k} \quad (6)$$

Obviously, this is converging to 1. If we consider the cumulated probability of '*not obtaining tails in a row*', this can be described -with some approximations- by *F(L)*:

$$F(L) = \sum_{k=1}^{\infty} 1 - 2^{-k} \quad (7)$$

This sum is diverging. With k going from 1 to *n*, we could rewrite it as:

$$F(L) = (1 - 2^{-1}) + .. + (1 - 2^{-n}) \ \textit{if } k \to \infty, n{=}\infty \ [9] \ F(L) = n - (2^{-1} + .. + 2^{-n}) = \infty$$

Since $(2^{-1} + \ldots + 2^{-n})$ is equal to [6] and converging to 1 and *n* is diverging, the series $F(L)$ [7] is diverging. This is another way to say that our less desirable events are indeed most likely to occur than the desired ones[8]. Quite fortunate payoff will instead occur very rarely with a probability converging to 0 (see also Fig. 1). The expected value is an "average" of the gain we could expect; however, many other elements should be taken into consideration. In fact, the expected value is giving us the mean coming from all the possible occurrences with different gain and related probability. Since we normally do not trust on average value concerning any kind of distribution but we want to get additional information (e.g. standard deviation, dispersion index, skewness, etc.), there would be similar reasons to also consider the variability of the gains and their connected probabilities. Actually, if we consider the original formulation of the St. Petersburg lottery, we can deduce that the EV is not infinite but undefined. In fact, since the variance is not finite anymore[9], the strong law of large number will not apply. In the game, the EV is influenced by the outstanding gain connected to very rare occurrences. Why should we therefore only focus on the EV of this variable and undermine that there would be a high gain only with sufficient high repetitions of the game? What are we neglecting to consider?

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[7] Starting from a payoff 2^1 with probability 2^{-1}.

[8] More and more fortunate events will occur less and less frequently while games that would generate a less desirable payoff will asymptotically stabilize to, at least, half of the total plays. The remaining n/2 plays would generate modest gains that would probably only balance the fees for entering the game

[9] If our variables are independent but not identically distributed then the average should converge to the EV $\bar{X}_n - E(\bar{X}_n) \to 0$ only if X_k has a finite variance: $\sum_{K=1}^{\infty} Var(X_k)/K^2 < \infty$ See *Kolmogorov's strong law of large numbers* and Sen; Singer (1993). *Large sample methods in statistics.* Ch. & Hall

6. Some behavioural remarks: could we better perceive negative or positive consequences? Are our choices based on a time context?

One important factor to consider in the decision-making is the animal nature and its reactions to external stimuli (space and time context). A recent neuroscience experiment[10] reveals that there are many factors pushing animals to take suboptimal choice for a large reward even if this is very rarely delivered. Among these factors, there are two important elements to consider: (1) their insensitivity to risk because they are not able to evaluate the uncertainty of the prize; (2) the perception of a "loss" as a punishment (they consider the loss as frequently omitted reward instead of a very probable occurrence). Another important element is the rigidity induced by the habit to conduct an apparently good strategy seeking the maximum reward (underestimating the risk), which biases the correct formation of risk aversion in case of very unsure reward. This factor is also correlated with the pure temptation to gamble which may be predominant over any other factors. The attraction to rewards can generate positive reinforcements dominating the risk of punishment. As concerns human gambling, an important key is "the risk of losing" (losing in a negative game session what was gained in a previous favourable game session[11]) that is different from the "failure to win" representing also the frustration caused by the lack of the expected gain. Focusing the attention on the "failure to win" only considers the frustration for missing an expected reward but does not take into account the risk of the negative events (in other words, the session game was considered misfortunate but not risky). The choice mechanism is therefore affected by many factors but obviously all these decisions are based on a finite segment of time in which the subjects reinforce a habit in order to reach his perceived optimal choice (we cannot therefore consider the context as an infinite space).

7. Are we neglecting to consider the non-ergodicity?

Recently, interesting observations were made on the expected value and its lack of capacity to determine the price of the Saint Petersburg game due to non-ergodic property of the time averages. The expected value formula implies that the time averages of the considered games are equal to the average of the entire system (ergodicity). Evaluating the ergodicity of a system is a very crucial element especially when conducting physics tests where the sample results of the experiments should generate reliable universal implications. In other words, ergodicity supposes that a system (probabilistic ensemble) observed for a sufficient

[10] Paglieri et al. 2014, *Nonhuman gamblers,* Frontiers in Behavioural Neuroscience (8:33).
[11] Zeeb et al., 2009.

period of time t is representative of all the possible states of the universe (sample space) in a way that the relative frequencies of selected sample coincide with the postulated predicted probabilities associated with the possible occurrences of the considered universe [11]. This means that we have to verify along with time our theoretical assumptions contained in the expected value formula [2]. In case the time averages coincide with ensemble average [10] the system is ergodic, otherwise it is a non-ergodic system.

$A = \langle A \rangle \quad (10)$ A is the time average and ⟨A⟩ the ensemble average. The consequence of ergodicity is that the variable of interest do not change overtime and even if very small fluctuations are observed, in a sufficiently long period, they do not influence the variables (this means that has not relevant effect on the ensemble system).

More generally, in continuous context, the condition of ergodicity can be defined as:

$$\lim_{t\to\infty} F\,(x,t) = P(x) \tag{11}$$

That is to say, that the relative frequencies F observed over time tends to the postulated probabilities P governing the ensemble system. In our case, the ensemble average can be described by the expected value defined by the Bernoulli game [2] while the time average is the average payoff coming from the experiments conducted over time [fig. 3]. The issue of ergodicity has been analysed in detail especially by Peters[12], and an immediate visualisation (proposed by Koelman[13] 2012) of the differences in averages can be useful to smoothly understand the context of this issue. If we consider a mechanism similar to the *Steinhaus Sequence*, we can build a recursive sequence based on the powers of 2 which follows a consequent recursive deterministic pattern. In the matrix, starting from the first row, each alternate sequence of empty cells is filled with the number corresponding to 2 powered to a number equal to the previous row number[14] (i.e. the empty cells of the 4th row are filled with 2^{4-1} and so on, see numbers in bold in Fig 2).

Fig. 2 - Building a matrix of the power of 2 using a *Steinhaus Sequence principle*

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2	.	*2*	.	*2*	.	*2*	..	.	*2*	*2*	.	*2*	.	*2*		..
2	***4***	*2*	.	*2*	***4***	*2*	..	*2*	***4***	*2*	.	*2*	***4***	*2*		..
2	*4*	*2*	***8***	*2*	*4*	*2*	..	*2*	*4*	*2*	***8***	*2*	*4*	*2*		..
2	*4*	*2*	*8*	*2*	*4*	*2*	***16***	*2*	*4*	*2*	*8*	*2*	*4*	*2*		..

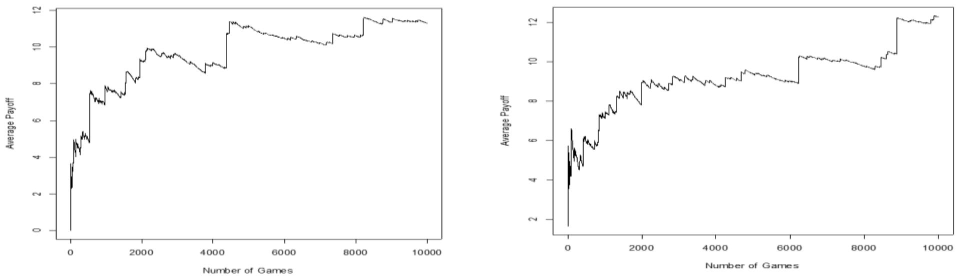
[12] Peters O. 2011, *The time resolution of the St. Petersburg Paradox*, Ph. Trans. Royal Society, n. 369

[13] This very useful hint was proposed by Johannes Koelman, *Statistical Physics Attacks St. Petersburg: Paradox Resolved*, on Science 2.0 (Scientific Blogging) 18th November 2012.

[14] This sequence is similar to the distribution of probability of the St. Petersburg game. In fact, if you casually choose a number from this matrix, you would have probability ½ to extract a 2, ¼ to extract a 4 and *1/n* to extract *n*. We could easily calculate the averages for the first *2, 3, 4..n* numbers located in the corresponding rows (the first 2 numbers in the 2nd row, the first 3 numbers in the 3rd row, etc.).

We can observe that the average is not independent (at least for a considerably large number of games) from the number of games played. These averages are finite and fluctuate considerably for each set of experimental games, although the pattern shows an infinite logarithmic increment. Since the average of all *time averages* does not converge to a finite value in the long run, the assumption of an ergodic context is confuted and time averages can not substitute the entire *space average* (as calculated with the expected value formula). Therefore, the non-ergodicity implies that time is a creative factor influencing the positive trend of time averages.

Fig. 3 – Two simulated sets of average payoff for 10,000 games



Source: author's simulation using R (for source code, see the references mentioned at the end of the article)

In order to establish a fair price, we could repeat the game n times and observe the simulated pattern of payoff; therefore, we could use the average payoff as an indicator of the general tendency and propose a price balanced on it (naturally, for each different set of repetition of the game the price would be different). The debate on non-ergodicity therefore becomes interesting if we consider the finite available time and if we are supposed to play only a limited number of games.

8. Probabilities to obtain a significant payoff

A significant gain should happen if a certain number of tails occur in a row. The main problem at decision level is to be aware of the probability that this event will happen and the respective probability that this would not happen (this is relevant because every time our suitable gain is not compensating our initial investment, we are facing a loss). However, more generally, if we consider that the probability to obtain n tails in a row is $P(n) = (2^{-1})^n$, it is immediate to recognise that a significant gain is based on a very low probability while the opposite and undesired events implying low gain or loss are likely to occur with a higher probability. The combination of expected gain in an infinite repetition of the game is biasing our decision because it lacks two main elements: the empirical limited duration of the game and its concrete management structure (time life resources that we are likely going to dedicate to this game and financial resources of the gambler and the bookmakers). Even if the probability of obtaining a significant number of tails in a row is only a partial consideration of the overall context of the game, this may be the main evidence that could lead to a wise decision in the short run, while the theoretical paradox is based on a supposed infinite context. Computer based simulation can easily confirm this kind of reasoning. Nowadays we dispose of free available tools to perceive the biases of the human expectations tending to over valuate fortunate events with low occurrence probability (see *references and further reading* section). The gamble booming in our society seems based, besides the psychological and sociological factors, on the incorrect way to calculate the occurrence of favourable events and the mechanism that generates them[15]. At the end of the XX century Ambrose Bierce said that lottery is a tax on people who are bad at mathematics. Nowadays this could be the case of only some gamblers, but it seems that most of them are quite aware of their systematic position of inferiority to the bookmakers even because of their empirical findings after some repeated bets. However, the complete scientific awareness would maybe reduce in a more aware and radical way the bias of the choices and not only as concerns gambling. Actually, in less than a century, we have suddenly awakened in a world where every tool we use is completely parametrised and probabilistically set. Setting a price for a plane ticket or booking a hotel room, sending customised commercial messages on internet tracking the users' behaviour, etc. These models certainly do not set infinite and asymptotic parameters when evaluating our daily life behaviours. In the next paragraph, we make some reflections on how this affects our lives.

9. Assessing the risk of everyday life decisions: learning about failure (ex-post adjustments) or log-frame evaluation (ex-ante and instant adjustments)?

Compared to the beginning of the XVIII century many things profoundly changed. The widespread awareness about general concepts related to probability and the computer literacy should switch the debate to a completely different level than a pure academic debate among scientists. Nevertheless, despite the application of the probability models and the computerisation of almost every structured human life management or decision process, we still face a society in which some decisions (extreme gambling, compulsive buying, etc.) are biased because of the lack of consideration about the risk assessment. This is part of the human nature which perceives the risk very differently according many different contexts, time

[15] A risk assessment literacy would not only be an awareness of the formal description of the phenomenon but especially a reasonable way to make some decisions.

and emotional reactions[16] (par. 6). The *Saint Petersburg paradox*, to a certain extent recalls the martingale mechanism but the inner process of the game (theoretical infinite gain) makes us focus more on the infinite expected value than all the other factors that in the medium-short run could affect our desired perspective of gain. However, also in this case we could demonstrate some inconsistency limiting our analysis to the available time for the gambler and the possible occurrences he would face with their consequences. Empirically and theoretically this would not be worth much in term of human resources dedicated to it. Nevertheless, if we consider other fields of application such as physics or computational models including patterns with almost infinite cycles, this analysis then assumes a different nature and it remains very useful because of the non-ergodic implications and their consequences. In this case, we consider a completely different structure and context than the initial framework connected to human gambling repeated to infinity. Recently the concept of *learning about failure* became very fashionable with a huge literature applying it to different fields of the social sciences; the empirical counterproof and the awareness of the positive expectation fallacy could bring some consciousness of the differences among theoretical and real effects of infinite expected value applied to different contexts. Nevertheless, in the context of extreme gambling, the only *learning about failure* could lead to very negative results because your failure would likely assume the form of a bankruptcy from which you could hardly learn and start over again avoiding your past errors. If someone should still have some scepticism, he would not to certainly put trust on famous empirical analogical experiments anymore (e.g. Buffon[17]). Nowadays you may want to empirically experience the consequences of playing a consistent number of times with simulated random variables games which are broadly available also on line (some references and example are indicated in the references and further reading section) or generate computed results with simple programming (e.g. java script or even setting parameters with simple spread sheets). Anyone experimenting the simulations would be happy about avoiding losing time and other resources on the research of empirical proofs.

[16] How would you feel about someone saying: '*in a martingale game, at some point, you will surely win whatever amount you desire, provided that you continue betting in order to reach a positive payoff (compensating all previous losses)'.* This is theoretically true only if you have infinite time and money.

[17] In 1777, Buffon conducted an experiment repeating it for 2048 times and found out that the number of tails in a row corresponding to 1, 2, 3, 4, 5, 6, 7, 8, and 9 had a frequency of 1061, 494, 232, 137, 56, 29, 25, 8, and 6 respectively. The average payoff was 4.91. *Essais d'arithmétique morale* in *Supplément* à l'*Histoire Naturelle*, V. 4, Imprimerie Royale, Paris. At page 394 Buffon describes the experiment he made with the help of a kid: *J'ai donc fait deux mille quarante-huit expériences sur cette question, c'est-à-dire j'ai joué deux mille quarantehuit fois ce jeu, en faisant jeter la pièce par un enfant.*

Indeed, since compulsive gambling is becoming a social problem, it could become a legal requirement for bookmakers to put free simulation machines at the disposals of the clients (foreseeing some symbolic form of incentive to test it before playing with real money), this would be more effective than a simple disclaimer only mentioning the risks leading to pathological gambling and the list of the probabilities connected to the potential gains. Finally, why is the study of this paradox still so important after two centuries? Because it has to do with the decision-making and risk assessment[18]. The considerations on the time factor and the non-ergodic context of the experiment reveal new useful elements for assessing social and scientific phenomena. This approach considers the pattern of payoff in finite plays, their related *time average* (par. 7) and the behavioural components of the choices (par. 6). The non-ergodic property of the St. Petersburg system underlines the correlation of the average payoff with the time factor and the misperception of the probability of possible negative events computed and attenuated in the *ensemble system.* However, all these factors have to be inserted in a variegated framework connected to human nature and all the elements that may affect the context. Therefore, an apparently purely mathematical problem is instead involving a very horizontal multidisciplinary approach. For that reason I acknowledge T. Parisi (Engineer), for the clues on IT programming and the important reflections on the capillary use of technology in our daily life. As concerns the psychological factors of choices, A. Carstoiu (medical doctor at *Clinic de Psihiatrie* in Bucharest), showed me the mechanism affecting human choice while S. Bosnic (researcher at Croatian Veterinary Institute) made me interesting comments on animal behaviours. I am also grateful to G. Celeste, E. Morandi and P. Pasqualis (directive members of the Italian Notariat) with whom we proposed very challenging research projects that allowed the knowledge exchange with Academics at the University of Moscow (HSE) and Saint Petersburg[19]. It seems therefore not a chance that in this last city the *Imperial Academy of Science* first published D. Bernoulli's work and his first proposed solution to solve the paradox. Indeed, in Russia, there was the opportunity to present some of our results at multidisciplinary conferences for the analysis of the economy and the society and we became keener on philosophical and conceptual aspects of the economic analysis. The paradox is one of the most challenging exercises because it involves

[18] Which are key factors of financial and insurance markets but, to some extent, with all kind of choices.

[19] Among them: the Scientific C. of Higher School of Economics of Moscow (V. Mkhitarian,V. Sirotin, M. Arkhipova and L. Rodionova), D. Raskov (St. Petersburg University), O. Ozerova (Sociological Inst., Russian Academy of Sciences, St. Petersburg), A. Nemtsov (Moscow Research Inst. of Psychiatry), M. Markov (St. Petersburg Univ.), J. Nye (George Mason Univ.), E. Poelmans (Univ. Leuven), K. Storchmann (NY Univ.), R. White (Univ. of Alabama) and J. Leitzel (Univ. of Chicago).

the capacity of rethinking about all the standards given for guaranteed in the scientific assessment. These arguments show that it is important to consider the irreversibility of the choices (in a given finite context), the behavioural factors, and the reasonable expected gain assessed from different point of view (considering the not-ergodic context and the EV capacity to properly describe the empirical outcome).

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SUMMARY

The *Saint Petersburg Paradox* is still a contemporary issue because of the great impact on the probabilistic theory and decision-making. This article proposes some hints on avoiding the trap of the infinite EV. The highly stochastic mechanism and its EV have always to be contextualized in the limited period where we take our choices taking into account all possible limitations deriving from the theory (including the non-ergodic features and some inappropriate consequences we may attribute to the EV). This *contextualisation* is one of the most important factors to consider especially when we deal with infinite quantity coming from models that may misrepresent our field of application and therefore generate paradoxes.

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